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Dear Family,

In Chapter 4, your child will learn about matrices and how to use them to solve linear systems.

A **matrix** is a rectangular array of numbers enclosed in brackets. A matrix is like a table without headings.

The numbers of rows and columns are called the **dimensions** of the matrix. A **square matrix** has the same numbers of rows and columns.

matrix: $A = \begin{bmatrix} 2 & 5 \\ 3 & -4 \\ 1 & 0 \end{bmatrix}$

dimensions: Matrix A is a 3×2 matrix.

address: The address of 3 is a_{21} .

Each value, or **entry**, in the matrix is named by an **address** that gives its row and column.

Several operations can be performed with matrices:

Operation	Requirements	How To	Example
Addition/ Subtraction	The matrices must have the same dimensions.	Add or subtract corresponding entries.	$\begin{bmatrix} 2 & 5 \\ 3 & -4 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ -8 & 9 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 2+3 & 5+6 \\ 3+(-8) & -4+9 \\ 1+2 & 0+5 \end{bmatrix}$
Scalar Multiplication	No requirements.	Multiply each entry by the same number, or scalar .	$4 \begin{bmatrix} 2 & 5 \\ 3 & -4 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 4(2) & 4(5) \\ 4(3) & 4(-4) \\ 4(1) & 4(0) \end{bmatrix}$
Matrix Multiplication	The <i>inside</i> dimensions must be the same. (The dimensions of the product will be the <i>outside</i> dimensions.)	Working with row i of the first matrix and column j of the second matrix, add the products of consecutive entries. The result goes in row i column j of the matrix product.	The dimensions are 3×2 and 2×2 . $\begin{bmatrix} 2 & 5 \\ 3 & -4 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 3 & 6 \\ -8 & 9 \end{bmatrix} =$ $\begin{bmatrix} 2(3) + 5(-8) & 2(6) + 5(9) \\ 3(3) + (-4)(-8) & 3(6) + (-4)(9) \\ 1(3) + 0(-8) & 1(6) + 0(9) \end{bmatrix}$ The entry at p_{32} is calculated from row 3 of the first matrix and column 2 of the second matrix.

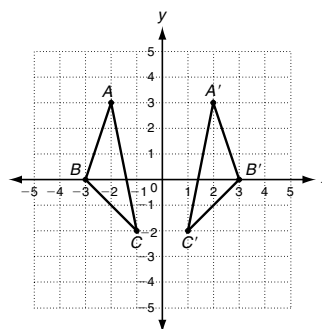
A **multiplicative identity matrix** is a square matrix with 1's along the **main diagonal** (top left to bottom right) and 0's for every other entry. The product of square matrix A and the identity matrix is simply matrix A . The product of a matrix and its **multiplicative inverse** is the identity.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 3 & -1 \end{bmatrix} \quad \begin{bmatrix} \frac{1}{14} & \frac{2}{7} \\ \frac{3}{14} & \frac{-1}{7} \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

A matrix can represent the x - and y -coordinates of the vertices of a polygon. You can then transform the polygon using matrix operations.

$$\Delta ABC: \begin{bmatrix} -2 & -3 & -1 \\ 3 & 0 & -2 \end{bmatrix} \begin{array}{l} \leftarrow x\text{-coordinates} \\ \leftarrow y\text{-coordinates} \end{array}$$

$$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ A & B & C \end{array}$$



$$\Delta A'B'C': \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & -3 & -1 \\ 3 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 0 & -2 \end{bmatrix}$$

Matrices can also represent a linear system in two ways:

Linear System	Matrix Equation	Augmented Matrix
$\begin{cases} 2x + 4y = 20 \\ 3x - y = 9 \end{cases}$	$\begin{bmatrix} 2 & 4 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 20 \\ 9 \end{bmatrix}$	$\left[\begin{array}{cc c} 2 & 4 & 20 \\ 3 & -1 & 9 \end{array} \right]$

Cramer's rule allows you to solve a linear system using determinants. A **determinant** is a single number calculated from the entries in the matrix.

determinant of coefficient matrix: $\det \begin{bmatrix} 2 & 4 \\ 3 & -1 \end{bmatrix} = (2)(-1) - (3)(4) = -14$

replace x -coefficients with constants: $\det \begin{bmatrix} 20 & 4 \\ 9 & -1 \end{bmatrix} = -56$, so $x = \frac{-56}{-14} = 4$

replace y -coefficients with constants: $\det \begin{bmatrix} 2 & 20 \\ 3 & 9 \end{bmatrix} = -42$, so $y = \frac{-42}{-14} = 3$

You can also solve a linear system with a matrix equation and the multiplicative inverse of the coefficient matrix.

matrix equation: $\begin{bmatrix} 2 & 4 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 20 \\ 9 \end{bmatrix}$

solution: $\begin{bmatrix} \frac{1}{14} & \frac{2}{7} \\ \frac{3}{14} & \frac{-1}{7} \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{14} & \frac{2}{7} \\ \frac{3}{14} & \frac{-1}{7} \end{bmatrix} \begin{bmatrix} 20 \\ 9 \end{bmatrix}$ $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$, so $x = 4$
 $y = 3$

A third way to solve a linear system uses **row operations** to change the augmented matrix into **reduced row-echelon form**.

augmented matrix: $\left[\begin{array}{cc|c} 2 & 4 & 20 \\ 3 & -1 & 9 \end{array} \right]$

reduced row-echelon form: $\left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & 3 \end{array} \right]$, so $x = 4$
 $y = 3$

All three methods can be extended to systems with more than two equations and two variables.

For additional resources, visit go.hrw.com and enter the keyword MB7 Parent.